CURRENT DISTRIBUTION OF PERMEABLE ELECTRODES IN THE PRESENCE OF THE HALL EFFECT IN A STREAM OF ELECTRICALLY CONDUCTING MEDIUM

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 35–41, 1966

The two-dimensional problem of the current distribution on the surface of permeable electrodes contiguous with a stream of incompressible medium with Hall effect is considered. An electrically conducting medium with the same physical properties as those of the main stream is pumped in (out) through the electrodes.

This problem was solved in [1] for one particular case when the electrodes are impermeable. It was established that due to the Hall effect in magnetohydrodynamic channels the current is distributed nonuniformly on the electrodes; for values of the Hall parameter of the order of several units or greater, the current flows into an isotropically conducting medium mainly from a small portion on the edge of the electrode. It was also noted that this phenomenon creates unfavorable conditions for the operation of electrodes in magnetohydrodynamic devices.

It is shown in what follows that the current distribution on the electrodes may be controlled, and in particular made more uniform, by injecting an electrically conducting medium.

1. Suppose we have an infinite plane magnetohydrodynamic channel with segmented electrodes with dimensions small compared to the width of the channel. Now, since the problem is to find the current distribution on the electrodes only, it is quite sufficient to investigate the behavior of the current on the electrodes of just one wall and neglect the influence of the other walls. Such an assumption does not introduce any appreciable error into the results and simplifies the solution of the problem considerably.

Making this simplification, we shall assume that the stream of conducting medium $\mathbf{v}(u(x,y),\ v(x,y),0)$, having isotropic conductivity, fills the lower half-plane, and that the real axis coincides with the direction of the wall (Fig. 1). The segments $a_k b_k$ (k = 1, 2, ..., p) in Fig. 1 indicate the portions of the electrodes which come in contact with the stream of medium, while the remaining portion of the real axis Ox represents the boundary of the insulating walls. It is assumed that the external magnetic field $\mathbf{H}(0,0,H_z)$ is uniform in the lower half-plane, and that the magnetic self-field of the currents considered in the medium is small and may be neglected.

The electrodes of the other wall of the channel are removed to an infinitely distant point.

We have the following system of equations for finding the currents in the lower half-plane:

$$j_x = \sigma\left(-\frac{\partial\varphi}{\partial x} + \frac{1}{c}vH_0\right) - \omega\tau j_y,$$

$$j_y = \sigma\left(-\frac{\partial\varphi}{\partial y} - \frac{1}{c}uH_0\right) + \omega\tau j_x,$$

$$\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad H_0 \equiv H_z. \quad (1.1)$$

Here $j_x(x,y)$, $j_y(x,y)$ are the components of the current vector, $\varphi(x,y)$ is the potential of the electrostatic field, σ is the electrical conductivity, $\omega\tau$ is the Hall parameter, and σ and $\omega\tau$ are constants.

Assuming that the properties of the materials of which the electrodes and insulating walls are made are perfect, we obtain the boundary conditions for which the system (1.1) must be solved,

$$j_x + \omega\tau j_y = \frac{1}{c}\sigma H_0 v(x), \quad a_k < x < b_k \quad \text{at } y = 0$$

$$j_y = 0, \qquad b_k < x < a_{k+1} \quad \text{at } y = 0$$

$$\begin{pmatrix} k = 1, 2, \ldots, p \\ a_{p+1} = a_1 \end{pmatrix}. \quad (1.2)$$

At infinity there are current sources or sinks.

On the assumptions which have been made it follows from system (1.1) that the electric current field is solenoidal and irrotational, and that consequently we may introduce the complex potential of the electric current F(z) by the formulas

$$j_x(x,y) = \frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}, \qquad j_y(x,y) = \frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x}$$

$$F(z) = P(x,y) + iQ(x,y) \qquad (z = x + iy)$$

$$\frac{dF(z)}{dz} = \frac{\partial P}{\partial x} + i\frac{\partial Q}{\partial x} = j_x(x,y) - ij_y(x,y),$$

$$\frac{dF(z)}{dz} = j(z) \quad (1.3)$$

In (1.3) P(x, y) is the potential function, and Q(x, y) is the electric current function; j(z) is the complex electric current.

Determining the currents in the half-plane Im z < < 0 now reduces to the following boundary problem for the complex current:

$$\operatorname{Re}\{(1 + i\omega\tau)j(x)\} = \frac{\sigma H_0}{c}v(x) \quad \text{on } L',$$

$$\operatorname{Im} j(x) = 0 \quad \text{on } L''. \quad (1.4)$$

Here L' indicates all the segments $a_k b_k$ (k = 1, ..., p), and L" the remaining part of the real axis.

In view of the initial assumptions j(z) is characterized at infinity by the expansion

$$j(z) = \frac{\gamma_1}{z} + \frac{\gamma_2}{z^2} + \ldots, \quad |z| \to \infty .$$

The boundary problem (1.4) may easily be solved by reducing it to a generalized linear boundary value problem [2, 3]. In order to do this we make an analytic continuation of the function j(z) into the upper-half plane and impose the condition that v(x) should satisfy a Hölder condition on L'. We shall designate the upper and lower half-planes by S^+ and S^-, and in passing around the real axis we shall take the positive direction to be that which leaves the region S^+ on the left.

From (1.4) we then obtain the following generalized linear boundary value problem (Riemann problem):

$$\Psi^+(x) = -\frac{1+i\omega\tau}{1-i\omega\tau}\Psi^-(x) + \frac{2\sigma H_0 v(x)}{c(1-i\omega\tau)} \quad \text{on } L',$$

$$\Psi^+(x) = \Psi^-(x) \quad \text{on } L'',$$

$$\Psi(z) = \begin{cases} \Psi^+(z) & \text{at } z\in S^+, \quad \Psi^+(z) = \overline{j(\bar z)}, \\ \Psi^-(z) & \text{at } z\in S^-, \quad \Psi^-(z) = j(z). \end{cases} \tag{1.5}$$

The solution of the boundary value problem (1.5), which vanishes at infinity, in the class of functions unbounded (but integrable) at the ends of the electrodes has the form

$$\Psi(z) = \prod_{k=1}^{p}(z-a_k)^{-1/2-\varepsilon}(z-b_k)^{-1/2+\varepsilon}\Big\{\frac{\sigma H_0}{\pi c\sqrt{1+\omega^2\tau^2}}\times$$
$$\times\int_{L'}\Big|\prod_{k=1}^{p}(x-a_k)^{1/2+\varepsilon}(x-b_k)^{1/2-\varepsilon}\Big|\frac{v(x)\,dx}{x-z} + C_1 z^{p-1} +$$
$$+ C_2 z^{p-2} + \ldots + C_p\Big\},$$
$$(\varepsilon = \pi^{-1}\operatorname{arc\,tg}\omega\tau, \quad 0 \leqslant \varepsilon < 1/2). \tag{1.6}$$

For the function under the product sign in (1.6) we choose the branch which is characterized for large $|z|$ by the expansion

$$\prod_{k=1}^{p}(z-a_k)^{-1/2-\varepsilon}(z-b_k)^{-1/2+\varepsilon} = \frac{1}{z^p} + \frac{d_{p+1}}{z^{p+1}} + \ldots \tag{1.7}$$
$$|z| \to \infty.$$

The components of the electric current j_x, j_y on the electrodes are found from the Sokhotskii-Plemel formula

$$j_x(x) = \frac{\Psi^+(x) + \Psi^-(x)}{2} =$$
$$= \frac{\sigma H_0 v(x)}{c(1+\omega^2\tau^2)} + \Big|\prod_{k=1}^{p}(x-a_k)^{-1/2-\varepsilon}(x-b_k)^{-1/2+\varepsilon}\Big|\times$$
$$\times\Big\{\frac{\omega\tau\sigma H_0}{\pi c(1+\omega^2\tau^2)}\int_{L'}\Big|\prod_{k=1}^{p}(t-a_k)^{1/2+\varepsilon}(t-b_k)^{1/2-\varepsilon}\Big|\frac{v(t)\,dt}{t-x} +$$
$$+\frac{\omega\tau(C_1x^{p-1}+C_2x^{p-2}+\ldots+C_p)}{\sqrt{1+\omega^2\tau^2}}\Big\}, \tag{1.8}$$

$$j_y(x) = \frac{\Psi^+(x) - \Psi^-(x)}{2i} =$$
$$= \frac{\omega\tau\sigma H_0 v(x)}{c(1+\omega^2\tau^2)} - \Big|\prod_{k=1}^{p}(x-a_k)^{-1/2-\varepsilon}(x-b_k)^{-1/2+\varepsilon}\Big|\times$$
$$\times\Big\{\frac{\sigma H_0}{\pi c(1+\omega^2\tau^2)}\int_{L'}\Big|\prod_{k=1}^{p}(t-a_k)^{1/2+\varepsilon}(t-b_k)^{1/2-\varepsilon}\Big|\frac{v(t)\,dt}{t-x} +$$
$$+\frac{C_1x^{p-1}+C_2x^{p-2}+\ldots+C_p}{\sqrt{1+\omega^2\tau^2}}\Big\}. \tag{1.9}$$

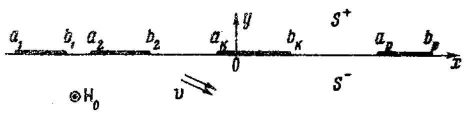

Fig. 1

The constants C_k ($k = 1, 2, \ldots, p$) are determined uniquely from the linear system of p equations

$$\int_{a_k}^{b_k} j_y(x)\,dx = I_k \quad (k = 1, 2, \ldots, p) \tag{1.10}$$

where I_k is the total current flowing through the k-th electode.

In the solution obtained j_x and j_y go to an infinity of order less than unity at the points a_k and b_k. Physically, these singularities indicate a concentration of currents at the ends of the electrodes and in their neighborhood. Other solutions may also be constructed which remain bounded close to any previously specified ends of the electrodes (where they necessarily vanish). In concrete cases the choice of the required solution is determined on the basis of additional physical conditions and assumptions. In particular, we must take into account the electrical circuit joining the electrodes, their relative positioning and dimensions, the law of normal velocity distribution on the electrodes, and other conditions. The general number of all possible solutions is equal to 2^{2p} [2, 3].

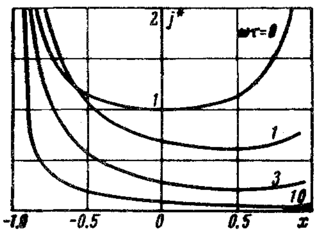

Fig. 2

2. We shall make a detailed examination of the particular case when a single electrode of finite dimensions

$$ab = 2l, \quad -l \leqslant x \leqslant l \text{ for } y = 0$$

is situated on the wall.

All solutions, the number of which is equal to 2^2, are now determined only by the conditions governing the injection of conducting fluid through the electrode.

We shall consider these solutions and deduce the conditions which the injection velocity at the electrode should satisfy in this case.

(a) Solution unbounded at both ends of the electrode. In the absence of pumping ($v(x) = 0$ on ab) a spreading out of the current flowing through the electrode in the region S^- will be accompanied by a current concentration at the ends of the electrodes a and b. The current distribution on the electrode is found on the basis of the solution already obtained (1.8), (1.9),

$$j_x(x) = \frac{\omega\tau C}{\sqrt{1+\omega^2\tau^2}}(l+x)^{-1/2-\varepsilon}(l-x)^{-1/2+\varepsilon}, \tag{2.1}$$

$$j_y(x) = \frac{-C}{\sqrt{1+\omega^2\tau^2}}(l+x)^{-1/2-\varepsilon}(l-x)^{-1/2+\varepsilon},$$
$$(-l \leqslant x \leqslant l). \tag{2.2}$$

The constant C is determined from the condition (1.10)

$$I = \frac{-C}{\sqrt{1+\omega^2\tau^2}}\int_{-l}^{l}(l+x)^{-1/2-\varepsilon}(l-x)^{-1/2+\varepsilon}\,dx =$$
$$= \frac{-C\Gamma(1/2-\varepsilon)\Gamma(1/2+\varepsilon)}{\sqrt{1+\omega^2\tau^2}} =$$
$$= \frac{-\pi C}{\cos\pi\varepsilon\sqrt{1+\omega^2\tau^2}}, \qquad C = -\frac{I}{\pi}. \tag{2.3}$$

Formulas (2.1)-(2.3) show the influence of anisotropic conductivity of the medium on the current distribution at the electrode. As the parameter $\omega\tau$ increases, the current density increases at the edge of the electrode near the end a and decreases correspondingly at the end b. This effect is illustrated in Fig. 2, where curves $j^* = \pi j_y(x)/I$ are constructed for four values of the parameter $\omega\tau = 0, 1, 3$ and 10, while for simplicity in the calculations it was assumed that $l = 1$.

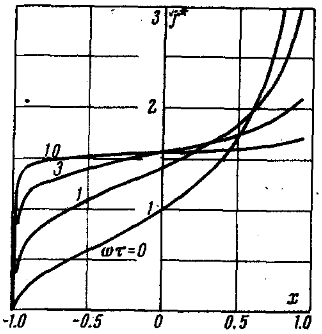

Fig. 3

If $v(x, 0) \neq 0$ at the electrode, then instead of (2.2) we have

$$j_y(r) = \frac{\omega\tau\sigma H_0 v(x)}{c(1+\omega^2\tau^2)} - \frac{\sigma H_0 (l+x)^{-1/2-\varepsilon}(l-x)^{-1/2+\varepsilon}}{\pi c(1+\omega^2\tau^2)} \times$$
$$\times \int_{-l}^{l} (l+t)^{1/2+\varepsilon}(l-t)^{1/2-\varepsilon} \frac{v(t)\,dt}{t-x} + \frac{I(l+x)^{-1/2-\varepsilon}(l-x)^{-1/2+\varepsilon}}{\pi\sqrt{1+\omega^2\tau^2}}$$
$$(-l \leqslant x,\ t \leqslant l). \qquad (2.4)$$

We omit the formula for the tangential current component at the electrode here and in what follows; if necessary it may easily be written down using the boundary condition (1.2) or the Sokhotskii-Plemel formula.

In accordance with (1.6), the current distribution next to the electrode is determined from the following expression:

$$j(z) = \frac{\sigma H_0 (z+l)^{-1/2-\varepsilon}(z-l)^{-1/2+\varepsilon}}{\pi c\sqrt{1+\omega^2\tau^2}} \int_{-l}^{l} (l+x)^{1/2+\varepsilon}(l-x)^{1/2-\varepsilon} \frac{v(x)\,dx}{x-z} -$$
$$-\frac{I}{\pi}(z+l)^{-1/2-\varepsilon}(z-l)^{-1/2+\varepsilon} \qquad (z \in S^-,\ -l \leqslant x \leqslant l). \qquad (2.5)$$

(b) Solution bounded close to end a and unbounded close to end b. In order that the current $j_y(x)$, determined by formula (2.4), should take bounded values close to end a of the electrode, including the point a, it is clear that the integral equation

$$\frac{1}{c}\sigma H_0 \int_{-l}^{l} \left(\frac{l+t}{l-t}\right)^{-1/2+\varepsilon} v(t)\,dt = I\sqrt{1+\omega^2\tau^2}$$
$$(-l \leqslant t \leqslant l) \qquad (2.6)$$

must be satisfied.

This relation determines how the conducting medium should be pumped in (or out) over the section ab as a function of the size and direction of the total current flowing through the electrode, and of other physical properties of the stream.

Further, taking into account the identity

$$\int_{-l}^{l} (l+t)^{1/2+\varepsilon}(l-t)^{1/2-\varepsilon}\frac{v(t)\,dt}{t-x} =$$
$$= (l+x)\int_{-l}^{l}\left(\frac{l+t}{l-t}\right)^{-1/2+\varepsilon}\frac{v(t)\,dt}{t-x} +$$
$$+ \int_{-l}^{l}\left(\frac{l+t}{l-t}\right)^{-1/2+\varepsilon} v(t)\,dt \qquad (-l \leqslant t \leqslant l) \qquad (2.7)$$

and the condition (2.6), we obtain from (2.4) the current distribution at the electrode and in its neighborhood

$$j_y(x) = \frac{\omega\tau\sigma H_0 v(x)}{c(1+\omega^2\tau^2)} -$$
$$-\frac{\sigma H_0 (l+x)^{1/2-\varepsilon}(l-x)^{-1/2+\varepsilon}}{\pi c(1+\omega^2\tau^2)}\int_{-l}^{l}\left(\frac{l+t}{l-t}\right)^{-1/2+\varepsilon}\frac{v(t)\,dt}{t-x},$$
$$(-l \leqslant x,\ t \leqslant l) \qquad (2.8)$$

$$j(z) =$$
$$= \frac{\sigma H_0 (z+l)^{1/2-\varepsilon}(z-l)^{-1/2+\varepsilon}}{\pi c\sqrt{1+\omega^2\tau^2}}\int_{-l}^{l}(l+x)^{-1/2+\varepsilon}(l-x)^{1/2-\varepsilon}\frac{v(x)\,dx}{x-z}$$
$$(-l \leqslant x \leqslant l,\ \ z \in S^-). \qquad (2.9)$$

It is clear that close to the end a of the electrode the current is bounded and at the point a itself takes the value zero.

To illustrate the formulas obtained we shall consider an example. We shall suppose that $v(x)$ maintains a constant value over the entire length of the electrode and introduce the notation

$$\frac{1}{c}\sigma H_0 v(x) = \Delta_a = \text{const}, \quad -l < x < l \quad \text{at} \quad y = 0. \qquad (2.10)$$

Then taking into account the fact that

$$\int_{-l}^{l}(l+t)^{-1/2+\varepsilon}(l-t)^{1/2-\varepsilon}\,dt =$$
$$= 2l\Gamma\left(\frac{1}{2}+\varepsilon\right)\Gamma\left(\frac{3}{2}-\varepsilon\right) = \frac{\pi l(1-2\varepsilon)}{\cos\pi\varepsilon} \qquad (2.11)$$

we find from (2.6) that

$$\Delta_a = I/\pi l(1-2\varepsilon). \qquad (2.12)$$

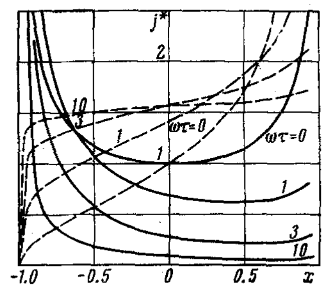

Fig. 4

The integrals in (2.8) and (2.9) are calculated with the help of Cauchy's theorem of residues. When the necessary transformations and calculations are carried out, we finally obtain

$$\Psi_a(z) = \frac{I}{\pi l(1-2\varepsilon)}\left[1-\left(\frac{z+l}{z-l}\right)^{1/2-\varepsilon}\right],$$
$$\Psi_a(z) = \begin{cases} \overline{j(\bar z)} = j_x + ij_y & \text{when } z \in S^+, \\ j(z) = j_x - ij_y & \text{when } z \in S^-. \end{cases} \qquad (2.13)$$

Hence

$$j_y(x) = \frac{\Psi_a^+(x) - \Psi_a^-(x)}{2i} = \frac{I(l+x)^{1/2-\varepsilon}(l-x)^{-1/2+\varepsilon}}{\pi l(1-2\varepsilon)\sqrt{1+\omega^2\tau^2}}$$
$$(-l \leqslant x \leqslant l). \qquad (2.14)$$

Curves of $j^* = \pi j_y(x)/I$ are constructed in Fig. 3 for four values of $\omega\tau = 0, 1, 3$ and 10 using this formula. Comparing these with similar curves in Fig. 2, it may be noted that when the conducting medium is pumped in, the given form of distribution of the normal current component at the electrode is evened out. This effect is strengthened as the parameter $\omega\tau$ increases, especially in those cases when the nonuniformity of the current distribution due to the anisotropy of the conductivity of the stream is particularly large. This is distinctly visible if Fig. 4 is considered, where the curves of j^* are represented by solid lines for case (a) when there is no pumping, and by dotted lines for case (b).

(c) Solution bounded close to end b and unbounded close to end *a*. All the discussions and calculations in this case are just the same as in the previous case, and so we shall give only the final formulas and confine ourselves to brief observations.

The condition which must now be obeyed by the injection rate at the electrode is as follows:

$$\frac{1}{c}\sigma H_0\int_{-l}^{l}\left(\frac{l+t}{l-t}\right)^{1/2+\varepsilon} v(t)\,dt = -I\sqrt{1+\omega^2\tau^2} \qquad (-l\leqslant t\leqslant l). \qquad (2.15)$$

The current distribution at the electrode and in its neighborhood is described by the formulas

$$j_y(x) = \frac{\omega\tau\sigma H_0 v(x)}{c(1+\omega^2\tau^2)} - \frac{\sigma H_0 (l+x)^{-1/2-\varepsilon}(l-x)^{1/2+\varepsilon}}{\pi c(1+\omega^2\tau^2)}\int_{-l}^{l}\left(\frac{l+t}{l-t}\right)^{1/2+\varepsilon}\frac{v(t)\,dt}{t-x} \qquad (-l\leqslant x, t\leqslant l), \qquad (2.16)$$

$$j(z) = \frac{-\sigma H_0 (z+l)^{-1/2-\varepsilon}(z-l)^{1/2+\varepsilon}}{\pi c\sqrt{1+\omega^2\tau^2}}\int_{-l}^{l}\left(\frac{l+x}{l-x}\right)^{1/2+\varepsilon}\frac{v(x)\,dx}{x-z} \qquad (-l\leqslant x\leqslant l,\ z\in S^-). \qquad (2.17)$$

Just as before, for the sake of an example, we shall suppose that the velocity at the electrode is constant along its entire length and we shall introduce the notation

$$\frac{1}{c}\sigma H_0 v(x) = \Delta_b = \text{const}, \quad -l\leqslant x\leqslant l \quad \text{at} \quad y=0. \qquad (2.18)$$

Then from (2.15)–(2.17) we obtain

$$\Delta_b = \frac{-I}{\pi l(1+2\varepsilon)}, \quad \Psi_b(z) = \frac{-I}{\pi l(1+2\varepsilon)}\left[1-\left(\frac{z-l}{z+l}\right)^{1/2+\varepsilon}\right], \qquad (2.19)$$

$$j_y(x) = \frac{I(l+x)^{-1/2-\varepsilon}(l-x)^{1/2+\varepsilon}}{\pi l(1+2\varepsilon)\sqrt{1+\omega^2\tau^2}} \qquad (-l\leqslant x\leqslant l). \qquad (2.20)$$

The curves for the current $j^* = \pi j_y(x)/I$ are given in Fig. 5. By examining them we see that pumping the conducting medium through the electrode at a constant velocity $v(x) = \Delta_b c/\sigma H_0$ achieves the opposite effect from that obtained previously. The nonuniformity of current distribution at the electrode increases, as is clear by comparing the curves in Figs. 2 and 5.

The different current distributions at a permeable electrode in the solutions given above are explained by the fact that the pumping in or out of conducting medium through the electrode along the channel wall induces an electromotive force which is directed either against the longitudinal Hall emf or in the same direction as it. In the first case there is compensation of the longitudinal Hall emf as a result of which the electric current flows more uniformly through the porous electrode than through a continuous electrode under the same influence of the Hall effect. This case corresponds to the solution considered in paragraph (b). In the opposite case the longitudinal Hall emf combines with the emf induced by the pumping, and the nonuniformity of current distribution along the electrode is enhanced. This result is obtained in paragraph (c).

(d) Solution bounded close to both ends of the electrode. In this solution the current density $j_y(x)$ must assume bounded values close to the two ends of the electrode *a* and b including the ends themselves. Clearly, conditions (2.6) and (2.15) must in this case be fulfilled simultaneously. Adding them, we obtain the single condition

$$\int_{-l}^{l}(l+t)^{-1/2+\varepsilon}(l-t)^{-1/2-\varepsilon}v(t)\,dt = 0 \qquad (-l\leqslant t\leqslant l). \qquad (2.21)$$

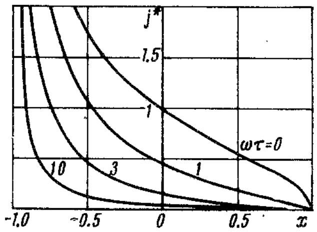

Fig. 5

The solution itself has the form

$$j_y(x) = \frac{\omega\tau\sigma H_0 v(x)}{c(1+\omega^2\tau^2)} - \frac{\sigma H_0 (l+x)^{1/2-\varepsilon}(l-x)^{1/2+\varepsilon}}{\pi c(1+\omega^2\tau^2)}\int_{-l}^{l}(l+t)^{-1/2+\varepsilon}(l-t)^{-1/2-\varepsilon}\frac{v(t)\,dt}{t-x} \qquad (-l\leqslant x,\ t\leqslant l) \qquad (2.22)$$

$$j(z) = \frac{-\sigma H_0 (z+l)^{1/2-\varepsilon}(z-l)^{1/2+\varepsilon}}{\pi c\sqrt{1+\omega^2\tau^2}}\int_{-l}^{l}(l+x)^{-1/2+\varepsilon}(l-x)^{-1/2-\varepsilon}\frac{v(x)\,dx}{x-z} \qquad (-l\leqslant x\leqslant l,\ z\in S^-). \qquad (2.23)$$

It follows immediately from (2.21) that it is impossible to obtain a current distribution bounded close to both ends of the electrode if $v(x)$ maintains a constant value over the whole length of the electrode. In all three conditions (2.6), (2.15), and (2.21) the normal velocity at the electrode appears in the integrand, and so for each of the solutions considered there are many functions $v(x)$ for which they are satisfied.

In conclusion we note the similarity which exists between the problems of the electrodynamics of a continuous medium which have been considered, and contact problems in elasticity theory. The current distribution along the electrode has the same form as the pressure distribution in an elastic body under a rigid die. We have in mind the plane contact problems of elasticity theory in the presence of frictional forces considered in monographs [4, 5].

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